1 Fix on Figure 3 in the Paper

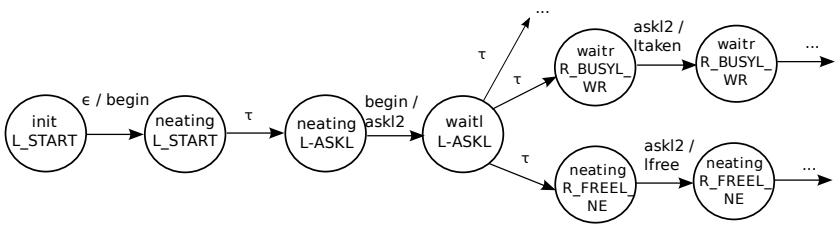

Fig. 1: Part of 1E_l

2 Proof for Theorem 2 in the Paper

Theorem 1. *Given a parameterized system with t different types of processes each defined using a set of behavioral automata* Prot*, the following holds for all Type I and II properties φ in the logic of $LTL\backslash X$*

$$\forall p \in [1,t] : \text{PATH}(1\text{E}_p) \subseteq \text{PATH}(sys(\bar{k}_t), S^{\bar{k}_t}) \Rightarrow \ \big(sys(\bar{k}_t) \models \varphi \Leftrightarrow sys(\bar{n}_t) \models \varphi\big)$$

where $\bar{n}_t = n_1, n_2, \ldots, n_t$, $\bar{k}_t = k_1, k_2, \ldots, k_t$, $S^{\bar{k}_t}$ is the set of states in $sys(\bar{k}_t)$.

Proof. We first prove the theorem for Type II properties. Recall from Section 4.2 that Type II property specification is concerned with actions of two processes directly communicating with each other (i.e. adjacent processes). Therefore, using Propositions 1, 2 and 3, it is required to prove that $\forall p_1, p_2 \in [1,t], \forall i_1 \leq n_1, \forall i_2 \leq n_2, \exists j_1 \leq k_1, j_2 \leq k_2$,

$$\text{PATH}(sys(\bar{n}_t)\downarrow\{i_1,i_2\}, S_I^{\bar{n}_t}) = \text{PATH}(sys(\bar{k}_t)\downarrow\{j_1,j_2\}, S_I^{\bar{k}_t})$$

such that i_1, i_2 and j_1, j_2 are adjacent processes in $sys(\bar{n}_t)$ and $sys(\bar{k}_t)$, respectively, and $S_I^{\bar{n}_t}$ and $S_I^{\bar{k}_t}$ are initial state-sets of $sys(\bar{n}_t)$ and $sys(\bar{k}_t)$ respectively.

Assume that there exists a sequence π of events in $\text{PATH}(sys(\bar{n}_t), S_I^{\bar{n}_t})$ such that $\pi\downarrow\{i_1,i_2\}$ is not present in $\text{PATH}(sys(\bar{k}_t)\downarrow\{j_1,j_2\}, S_I^{\bar{k}_t})$ for any $j_1 \leq k_1$ or $j_2 \leq k_2$. I.e., $\exists i_1 \leq n_1, i_2 \leq n_2, \forall j_1 \leq k_1, j_2 \leq k_2 : \text{PATH}(sys(\bar{n}_t)\downarrow\{i_1,i_2\}, S_I^{\bar{n}_t}) \neq \text{PATH}(sys(\bar{k}_t)\downarrow\{j_1,j_2\}, S_I^{\bar{k}_t})$. This assumption implies that (using Equations 1 and 2)

$$\begin{aligned} F_1(\pi\downarrow\{i_1,i_2\}, \textstyle\bigcup_{j_1,j_2} \text{PATH}(sys(\bar{k}_t)\downarrow\{j_1,j_2\}, S_I^{\bar{k}_t})) = \chi_1 \neq \emptyset \\ \Rightarrow \forall \pi'_1 \in \chi_1 : \exists e_1/e_0 : F_2(\pi\downarrow\{i_1,i_2\}, \pi'_1) = e_1/e_0 \end{aligned} \tag{1}$$

This, in turn, implies two possibilities as explained below:

Case 1. $e_1 = \epsilon$. In this case, e_1/e_0 is an autonomous move of process of type p_1 or p_2. As such a move is absent in all paths in $sys(\bar{k}_t)$, we can conclude that either $\text{PATH}(1\text{E}_{p_1}) \not\subseteq \text{PATH}(sys(\bar{k}_t), S^{\bar{k}_t})$ or $\text{PATH}(1\text{E}_{p_2}) \not\subseteq \text{PATH}(sys(\bar{k}_t), S^{\bar{k}_t})$.

Case 2. In the sequence π, process i_1 of type p_1 provides e_2/e_1 that resulted in e_1/e_0 in process i_2 of type p.

If e_2/e_1 followed by e_1/e_0 is absent in all paths in $\text{PATH}(sys(\bar{k}_t), S_I^{\bar{k}_t})$, we can immediately conclude that $\exists p' \in [1,t] : \text{PATH}(1\text{E}_{p'}) \not\subseteq \text{PATH}(sys(\bar{k}_t), S^{\bar{k}_t})$.

On the other hand, if e_2/e_1 followed by e_1/e_0 is present in some paths in $\text{PATH}(sys(\bar{k}_t), S_I^{\bar{k}_t})$, we can conclude that these do not appear (in projected form) in χ_1 (Equation 1). The projection of these paths mimic shorter prefixes of $\pi{\downarrow}\{i_1, i_2\}$ (see Definition of F_1 in Equation 2). If such a path (shorter than π') fails to match π on some e_1'/e_0' of the j'-th process of type p', the above arguments can be repeated for such a mismatch. I.e., if $e_1' = \epsilon$, $\text{PATH}(1\text{E}_{p'}) \not\subseteq \text{PATH}(sys(\bar{k}_t), S^{\bar{k}_t})$; otherwise, even shorter paths that mismatches π can be obtained by applying the arguments in Case 2 until only Case 1 is applicable and it follows that $\exists p \in [1,t] : \text{PATH}(1\text{E}_p) \not\subseteq \text{PATH}(sys(\bar{k}_t), S^{\bar{k}_t})$. This concludes the proof for Type II properties.

TYPE I PROPERTY. The proof for type I properties follows similar arguments as provided above.

We now proved that

$$\forall p \in [1,t] : \text{PATH}(1\text{E}_p) \subseteq \text{PATH}(sys(\bar{k}_t), S^{\bar{k}_t}) \Rightarrow \\ \text{PATH}(sys(\bar{n}_t){\downarrow}R, S_I^{\bar{n}_t}) = \text{PATH}(sys(\bar{k}_t){\downarrow}R, S_I^{\bar{k}_t})$$

where R is either one process (for type I properties) or two adjacent processes (for type II properties). Therefore, as shown in Proposition 2, all possible sequence of states of adjacent processes are identical in $sys(\bar{k}_t)$ and $sys(\bar{n}_t)$, therefore $\text{PATH}(sys(\bar{n}_t){\downarrow} R, S_I^{\bar{n}_t}) = \text{PATH}(sys(\bar{k}_t){\downarrow}R, S_I^{\bar{k}_t}) \Rightarrow sys(\bar{k}_t) \models \varphi \Leftrightarrow sys(\bar{n}_t) \models \varphi$ for all properties φ defined over actions of processes in R.